Performance Based Learning and Assessment Task

Getting Around Millbrook

I. **ASSESSSMENT TASK OVERVIEW & PURPOSE:**
Distance/Pythagorean Theorem. Students will discuss different ways to measure distance, and will conclude with an introduction to the Pythagorean Theorem.

II. **UNIT AUTHOR:**
Benjamin Ratliff, Millbrook High School, Frederick County

III. **COURSE:**
Geometry

IV. **CONTENT STRAND:**
Geometry, Measurement

V. **OBJECTIVES:**
Students will measure distance using both taxicab geometry and distance formula, create a graph and label points, find distance between two points, and derive the Pythagorean Theorem.

VI. **REFERENCE/RESOURCE MATERIALS:**
- Map of Millbrook High School, labeled (attached)

VII. **PRIMARY ASSESSMENT STRATEGIES:**
Assessment will be based on the attached rubric. Students will be given the opportunity to self-assess before the teacher assesses the student's work.

VIII. **EVALUATION CRITERIA:**
Students will be evaluated based on the attached rubric.

IX. **INSTRUCTIONAL TIME:**
This activity should take one 90-minute block or two 45-minute blocks.

Getting around Millbrook

Strand
Geometry/Measurement

Mathematical Objective(s)
After this activity, students will be able to find distance using both taxicab and Euclidean measures for distance, and will have a basic understanding of the Pythagorean Theorem. Students will practice drawing a graph, labeling points, and using the distance formula for finding the distance between those two points.

Related SOL

G.3 (distance formula), G.8 (Pythagorean Theorem), G.11 (solve real-world problems with circles)

NCTM Standards List all applicable NCTM standards related to each task/activity. Example:

- analyze properties and determine attributes of two- and three-dimensional objects
- use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations;
- investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates.
- draw and construct representations of two- and three-dimensional geometric objects using a variety of tools
- use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.

Materials/Resources
Classroom set of graphing calculators, ruler, compass, map of Millbrook (labeled)

Assumption of Prior Knowledge
Prior to this activity, students must be familiar with the distance formula. Students also should have some familiarity with a Cartesian coordinate plane. This activity is a good introduction to Pythagorean Theorem for an Honors level or advanced Academic level student. Students who still struggle with the concepts of distance formula, or have trouble with basic shapes (only on level 1 of the Van Hiele scale) may struggle with this activity.

Students also must have a familiarity with Millbrook High School (or your school, if you choose to adapt this lesson to your own map). Students also must realize that wireless internet signals can travel through walls (this information is explicitly stated in the activity).

Introduction: Setting Up the Mathematical Task

In this task, we will investigate different ways to measure distance in the context of a problem, and will learn about the Pythagorean Theorem for right triangles. The investigation will be split into 5 sections with a bonus investigation:

1. Using basic knowledge of shapes to find missing sides of rectangles
2. Using taxicab geometry to find walking distances around Millbrook
3. Using a Cartesian plane to label points within Millbrook
4. Using these points to find linear (Euclidean) distance between two points
5. Creating a triangle, and showing the Pythagorean Theorem from these distances.

   Bonus Investigation: create circles to show wireless internet coverage areas

To introduce the task, I will start a simple discussion about how hard it is to get around Millbrook: "Wow, my room is so far away from the cafeteria! How do you guys get there? If you had to get to the middle hallway, how would you get there?" This should get students thinking about how we measure distance in taxicab geometry. I want this introduction to be as free-flowing and informal as possible so that students feel comfortable and confident with the situation being presented. The introduction will be a whole-group discussion, with students participating as they choose.

Student Exploration

Next, I will give students a copy of the Millbrook map that I have labeled with points and some distances. The first activity will be a small group (pairs or sets of three students) activity in which students fill in the missing distances using basic knowledge of rectangles and the segment addition postulate. I will ensure that students have the correct completed map before proceeding. Even though students will be working together, each student should have his/her own map and activity sheet, since groups will be changing throughout this activity.

For the second section, the students will remain in their same groups. This section will ask students to think of distance as the total distance walked from one point to another (taxicab). This distance does not follow the distance formula, since it isn't a straight line. As such, there could be multiple paths between two points which are the shortest distance. Students will be able to recognize this by the problems and discussion with his/her partner. The teacher should facilitate any further discussion within the small groups.

For the third section, students will find a different partner. The teacher will model how to create the graph on the board, and show students how to find the example point. From there, the students will label each point asked on the activity sheet. They will then confirm their answers with their partners.

For the fourth section, the students will stay in their same groups. The students will read about the distance formula and apply it to the given sets of points. The teacher will only offer assistance if both or all group members are struggling. After a reasonable amount of time, the teacher will combine members of several pairs or small groups to create bigger groups. In these groups, the students will confirm their distances are correct, and discuss why this method of finding distance is different from the previous questions.

Once the students have completed through section Part 4 together, the teacher will hold a group discussion, both to make sure that the students have a good understanding of the differences between the two distances and to make sure that the students' computations are correct.

From here, the teacher will allow students to work individually for a short amount of time (10 minutes or so) to allow students to try section 5 (Pythagorean Theorem). Since the students have never seen this theorem, they may struggle with this concept. The teacher will then put students in medium-sized groups (4-6 students) to discuss what they think could be a general formula. After a short time (5-10 minutes), the teacher will then lead a whole-group discussion about the Pythagorean Theorem.

For Part 6, the teacher should follow a similar format to ask students to determine a general formula for taxicab geometry. As before, students should start in medium-sized groups to get some idea, then brought together for a whole-group discussion about the distance formula for taxicab geometry. If at all possible, do NOT lead students to the use of absolute value to make all distances positive.

Students will then receive a copy of the rubric. Since the students have not been given all of the answers, they will have a chance to self-assess their confidence with the material. The teacher will also grade their work to check the student's knowledge and meta-knowledge of the material presented.

For the final section, the teacher should be as uninvolved as possible. The last section asks students to take their own previous knowledge about circles to create coverage areas for wireless signals. It is for students who completed the material easily and wish for more challenging options. As such, the teacher should only be available to guide, not answer questions.

As a final summary of the day, the teacher should ask students to write a few sentences answering the following prompt:

“How is distance measured in Euclidean geometry? Give an example besides today's activity where that measurement does not accurately represent the distance in the situation. What is the Pythagorean Theorem? Give an example of a problem that can be solved by using the Pythagorean Theorem.”

Assessment List and Benchmarks

Students will be assessed using the attached rubric. The journal prompt to summarize the class is given above. The journal will be assessed as followed:

3: Student's gave an accurate summary of distance formula as a formula and a concept, and made a connection to the Pythagorean Theorem.

2: Student's gave an accurate summary, but did not draw connections between topics

1: Student's summary is incomplete, but does demonstrate some knowledge of topics

0: Student's summary is both incomplete and incorrect. Little or no demostration of correct knowledge of topics.

Getting around Millbrook High School

Remember your first day as a freshman here at Millbrook? Were you intimidated by the confusing layout of the school? How about how the room numbers don't make a lot of sense?!? Hopefully, after some experience, you have a good idea about the layout of Millbrook, and could help another newcomer feel more comfortable.

Part 1 – Intersections and Distances

Take a look at the Millbrook map you have. Notice that each intersection is labeled with a letter. Also notice that the main office (W) and Mr. Ratliff's room (Y) are labeled even though they are not intersections.

Take a look at the distances that are labeled between some of the points. Notice how some of the distances are missing? Your first task is to work with your partner to fill in any missing information. Do not measure the distances with a ruler; instead, try to find a matching side on a shape that you recognize.

Once you have filled in all of the missing distances, raise your hand so that I can check your work.

Part 2 – Walking

Adam, Bob, Charlie, and David like to hang out by the math office (A) after school each day. One day, they begin discussing how best to exit the school through the front door. All of the students think they know the best way, but they aren't sure how to prove it.

- Adam's path is AGX
- Bob's path is AQX
- Charlie's path is ABHPX
- David's path is ADKLPX

1. Assuming that the four boys all travel at the same speed, who will arrive at the front door first?

2. How long was each boy's path? Who traveled the furthest distance?

3. Find another path that is the same length as David's.

4. Mr. Ratliff's room is point Y. He needs to go to the office. What is the best path for him to take?

5. Is there another path that Mr. Ratliff could take that is the same distance? Why or why not?

6. One day Eric decides to hang out with Adam, Bob, Charlie, and David. Eric swears he knows a shortcut to the front door: he sneaks through the library (NV). Is he right? Why or why not?

Part 3 – Coordinate Geometry

Create a set of x and y axes with the origin at point Q. We are going to now use our map to label some of the points as a coordinate (x,y). For example, the point R could be written as the coordinate (80,0).

7. Label each of the following points as an ordered pair of coordinates

S	A
Y	U
C	K
G	O

Part 4 – Distance Formula

Recall that the distance between two points is given by the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

8. Find each of the following distances

YU	CK
GO	AX

9. Is the distance AX you found in problem 8 different than your answer in problem 2? If so, why, since both problems have the same beginning and end points?

Part 5 – Pythagorean Theorem

Look at the points A and R. Draw the distance between these two lines if:

a) You have to walk from point A to R

b) You use the distance formula from A to R

10. What shape did you make? Label each side with the correct length.

11. Notice that the distance formula can now be written as

$$d = \sqrt{220^2 + 80^2}$$

If I wanted to remove the radical (square root), what formula would I have?

12. Create a general formula to find the distance between two points if you know the legs of a right triangle.

Part 6 – Taxicab Distance

13. Look back to Part 2. Adam, Bob, Charlie, David, and Eric all walked different paths, but ended up traveling the same distance. Using coordinates, explain why this is true.

14. Using coordinates, create a general formula to find the distance between two points using taxicab geometry. Use this formula to check your answer from problem #4 to make sure it works correctly. REMEMBER: All distances must be positive! How can you make sure your distances are always positive?

Summary: Journal Entry

In a few sentences, discuss these questions:

How is distance measured in Euclidean geometry? Give an example besides today's activity where that measurement does not accurately represent the distance in the situation. What is the Pythagorean Theorem? Give an example of a problem that can be solved by using the Pythagorean Theorem.

Bonus Investigation – Wifi Coverage

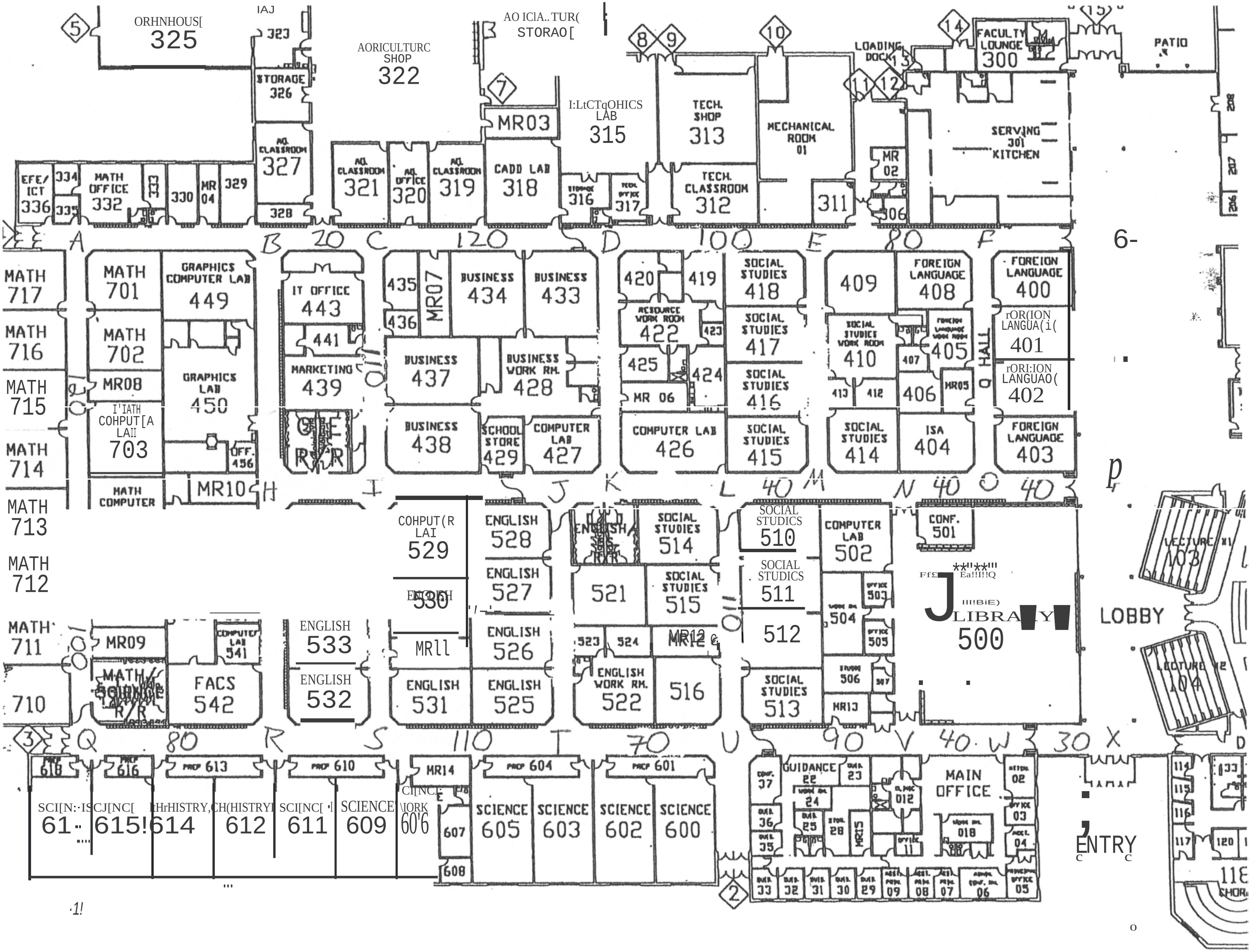
Wireless internet coverage comes from devices called routers. These routers broadcast the wireless signal in every direction, even through walls. However, these routers only have a limited range.

13. What shape is the coverage area of a router?

14. What tool could I use to draw this on my map?

15. Suppose I knew my router had a range of 200 feet. If I put this router at point M, what other points could receive service from this router?

16. The school is buying routers with a range of 200 feet, and needs to place them throughout the school in order to create a wireless network. Obviously, the school wants to buy as few routers as possible. Where could you place your routers so that all of the points were inside at least one router's coverage? What is the fewest number of routers you must buy?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SERVING 301 KITCHEN
MR 02
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TECH. CLASSROOM 312
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CADD LAB 318
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OFF. 456
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IT OFFICE 443
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MARKETING 439
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BUSINESS 434
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BUSINESS 437
BUSINESS WORK RM. 428
BUSINESS 438
SCHOOL STORE 429
COMPUTER LAB 427
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RESOURCE WORK ROOM 422
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SOCIAL STUDIES 418
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MR13
CONF. 501
LIBRARY 500
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LECTURE 103
LECTURE 104
ENTRY
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<u>Getting around Millbrook High School – Rubric</u>

The rubric will consist of two parts: How well you feel that you understand the material, and Mr. Ratliff's grade of your work. Here is how you should score yourself, and how Mr. Ratliff will score you:

Score	My self-score	Mr. Ratliff
3	I completely understand the problem, and am confident my answer is correct.	Concept is completely demonstrated, computation/answer is correct
2	I somewhat understand the problem, but am confident that my answer is correct.	Concept is somewhat demonstrated, but computation/answer is correct.
1	I somewhat understand the problem, but am unconfident with my answer	Concept is somewhat demonstrated, with errors in computation/answer
0	I do not understand the problem.	No understanding of concept present, computations/answer lacking

Rubric

#	Question	My score	Mr. Ratliff's score
1	Assuming that the four boys all travel at the same speed, who will arrive at the front door first?		
2	How long was each boy's path? Who traveled the furthest distance?		
3	Find another path that is the same length as David's.		
4	Mr. Ratliff's room is point Y. He needs to go to the office. What is the best path for him to take?		
5	Is there another path that Mr. Ratliff could take that is the same distance? Why or why not?		
6	One day Eric decides to hang out with Adam, Bob, Charlie, and David. Eric swears he knows a shortcut to the front door: he sneaks through the library (NV). Is he right? Why or why not?		
7	Label each of the following points as an ordered pair of coordinates		
8	Find each of the following distances		
9	Is the distance AX you found in problem 8 different than your answer in problem 2? If so, why, since both problems have the same beginning and end points		
10	What shape did you make? Label each side with the correct length.		
11	If I wanted to remove the radical (square root), what formula would I have		
12	Create a general formula to find the distance between two points if you know the legs of a right triangle.		
13	Look back to Part 2. Adam, Bob, Charlie, David, and Eric all walked different paths, but ended up traveling the same distance. Using coordinates, explain why this is true.		
14.	Using coordinates, create a general formula to find the distance between two points using taxicab geometry. Use this formula to check your answer from problem #4 to make sure it works correctly. REMEMBER: All distances must be positive! How can you make sure your distances are always positive?		

Getting around Millbrook High School - Key

Part 1 – Intersections and Distances

Take a look at the Millbrook map you have. Notice that each intersection is labeled with a letter. Also notice that the main office (W) and Mr. Ratliff's room (Y) are labeled even though they are not intersections.

Take a look at the distances that are labeled between some of the points. Notice how some of the distances are missing? Your first task is to work with your partner to fill in any missing information. Do not measure the distances with a ruler; instead, try to find a matching side on a shape that you recognize.

Once you have filled in all of the missing distances, raise your hand so that I can check your work.
See Attached Map Key

Part 2 – Walking

Adam, Bob, Charlie, and David like to hang out by the math office (A) after school each day. One day, they begin discussing how best to exit the school through the front door. All of the students think they know the best way, but they aren't sure how to prove it.

- Adam's path is AGX
- Bob's path is AQX
- Charlie's path is ABHPX
- David's path is ADKLPX

1. Assuming that the four boys all travel at the same speed, who will arrive at the front door first?

All would arrive at same time, since all travel 660 feet.

2. How long was each boy's path? Who traveled the furthest distance?

All boys traveled equal distance of 660 feet.

3. Find another path that is the same length as David's.

Multiple Answers: example: ABHISX

4. Mr. Ratliff's room is point Y. He needs to go to the office. What is the best path for him to take?

Only one correct answer: YQW. Any other path is longer than this path (510 feet).

5. Is there another path that Mr. Ratliff could take that is the same distance? Why or why not?

No, every other possible path is more than 510 feet.

6. One day Eric decides to hang out with Adam, Bob, Charlie, and David. Eric swears he knows a shortcut to the front door: he sneaks through the library (NV). Is he right? Why or why not?

No, distance is still 660 feet even using Eric's "shortcut".

Part 3 – Coordinate Geometry

Create a set of x and y axes with the origin at point Q. We are going to now use our map to label some of the points as a coordinate (x,y). For example, the point R could be written as the coordinate (80,0).

7. Label each of the following points as an ordered pair of coordinates

S	100 , 0	A	0 , 220
Y	0 , 100	U	280 , 0
C	100 , 220	K	220 , 110
G	440 , 220	O	400 , 110

Part 4 – Distance Formula

Recall that the distance between two points is given by the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

8. Find each of the following distances

YU	$\sqrt{88400} = 20\sqrt{221} \approx 297.32\ ft$	CK	$\sqrt{26500} = 10\sqrt{265} \approx 162.79\ ft$
GO	$\sqrt{13700} = 10\sqrt{137} \approx 117.05\ ft$	AX	$\sqrt{242000} = 20\sqrt{605} \approx 491.93\ ft$

9. Is the distance AX you found in problem 8 different than your answer in problem 2? If so, why, since both problems have the same beginning and end points?

Problem 2 is taxicab geo (can't go through walls), but Problem 8 is Euclidean (goes straight through walls)

Part 5 – Pythagorean Theorem

Look at the points A and R. Draw the distance between these two lines if:

a) You have to walk from point A to R

b) You use the distance formula from A to R

10. What shape did you make? Label each side with the correct length. Right triangle with height 220 and base 80.

11. Notice that the distance formula can now be written as

$$d = \sqrt{220^2 + 80^2}$$

If I wanted to remove the radical (square root), what formula would I have?

$$d^2 = 220^2 + 80^2$$

12. Create a general formula to find the distance between two points if you know the legs of a right triangle.

Pythagorean Theorem: $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$

Part 6 – Taxicab Distance

13. Look back to Part 2. Adam, Bob, Charlie, David, and Eric all walked different paths, but ended up traveling the same distance. Using coordinates, explain why this is true.

All begin and end at the same coordinates.

14. Using coordinates, create a general formula to find the distance between two points using taxicab geometry. Use this formula to check your answer from problem #4 to make sure it works correctly. REMEMBER: All distances must be positive! How can you make sure your distances are always positive?

$$d = |x_2 - x_1| + |y_2 - y_1|$$

Summary: Journal Entry

In a few sentences, discuss these questions:

How is distance measured in Euclidean geometry? Give an example besides today's activity where that measurement does not accurately represent the distance in the situation. What is the Pythagorean Theorem? Give an example of a problem that can be solved by using the Pythagorean Theorem.

Bonus Investigation – Wifi Coverage

Wireless internet coverage comes from devices called routers. These routers broadcast the wireless signal in every direction, even through walls. However, these routers only have a limited range.

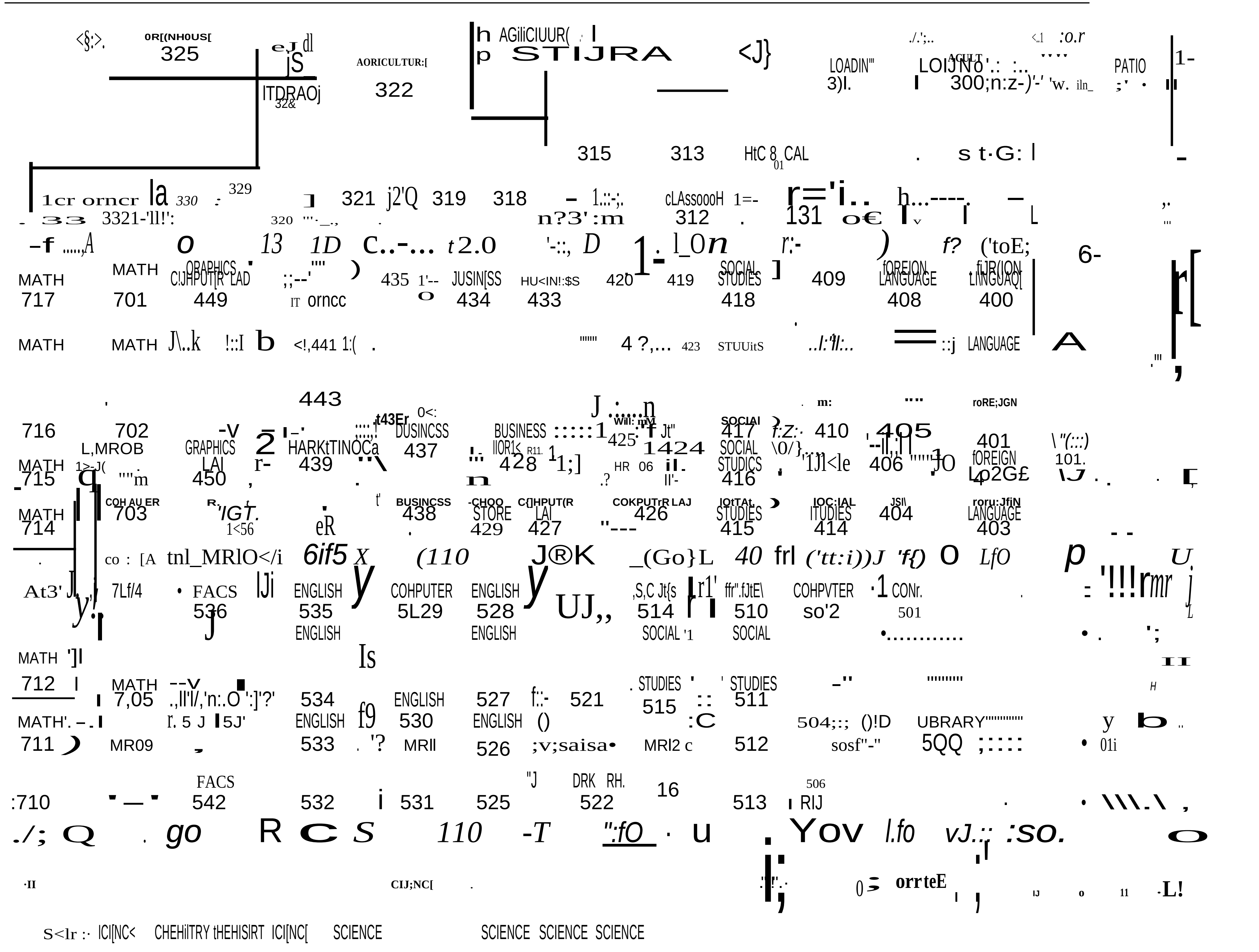
13. What shape is the coverage area of a router?
    Circle

14. What tool could I use to draw this on my map?
    Compass

15. Suppose I knew my router had a range of 200 feet. If I put this router at point M, what other points could receive service from this router?
    Measure 200 feet (distance AQ minus distance HI), put compass on point M, draw circle, see what other points are within circle.

16. The school is buying routers with a range of 200 feet, and needs to place them throughout the school in order to create a wireless network. Obviously, the school wants to buy as few routers as possible. Where could you place your routers so that all of the points were inside at least one router's coverage? What is the fewest number of routers you must buy?

Student discovery: See what they come up with!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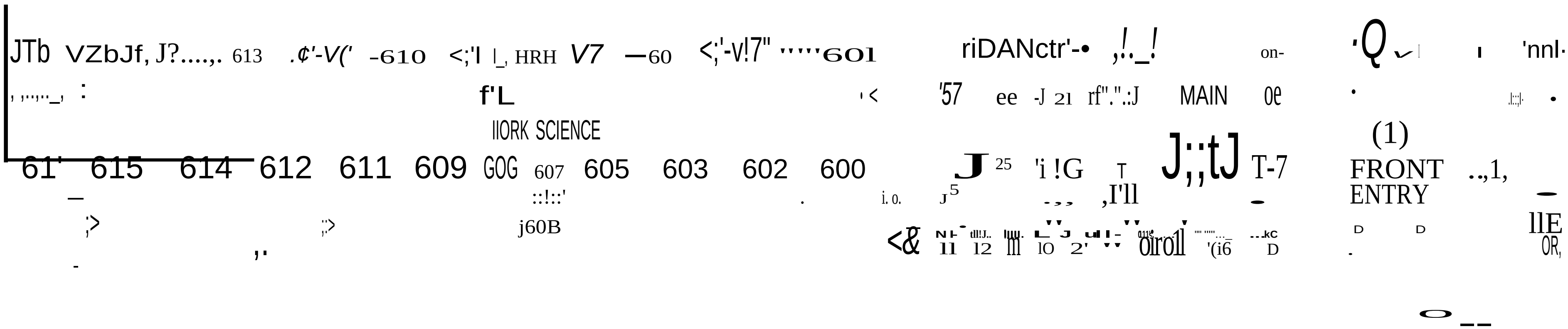

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